

STATEMENT OF TEACHING

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As an instructor of mathematics, I strive to treat my students with grace and dignity. I structure my courses to develop mathematical understanding and problem-solving skills while fostering critical thinking, mathematical creativity, confidence, patience, and resilience. I use collaborative, active-learning activities to build community and support a student body with a wide range of backgrounds and lived experiences. I also introduce students to the human aspects of mathematics, including its philosophy, culture, and history. Beyond the classroom, I mentor students through directed reading programs and engage broader audiences through public expository talks and other outreach activities. I view teaching, mentorship, and outreach as paramount academic responsibilities from which I leave a direct, tangible contribution to society.

1. GRACE IN TEACHING

I will begin not with my experiences as a teacher, but as a student. Entering Harvey Mudd College, I was a bright-eyed, enthusiastic mathematics student who'd previously attended many mathematics summer camps and performed well in math competitions. However, for a plethora of reasons, I struggled: my grades were mediocre and I did not participate in extracurricular mathematics activities many of my peers did.

Towards the end of my third year, I was facing a summer with nothing to do, and felt I needed to turn things around. I walked into my advisor, Arthur Benjamin's office, on a whim, and sheepishly asked him if he had any research projects for me. This was an out of the blue request, and Art could have easily turned me away. But he didn't - he presented me with a few possible questions right there, and offered to help me through the process of research on his own time. As the summer progressed, I'd spend my spare time thinking intensely about the question, and in a few months, I'd cooked up a full proof. This became my very first publication, and years later, it still remains one of my proudest works, as it represented a new beginning.

In Francis Su's essay "The Lessons of Grace in Teaching," he writes:

This is the Lesson of GRACE: Your accomplishments are NOT what make you a worthy human being. You learn this lesson by receiving GRACE: good things you didn't earn or deserve, but you're getting them anyway. ...

Sure, good instructional techniques are necessary for good teaching. But they are not sufficient. They are NOT the foundation. Grace-filled relationships with your students are the foundation for good teaching, because it gives you freedom to explore, freedom to fail. Freedom to let students take control of their own learning, freedom to affirm the struggling student by your own weakness. Grace amplifies the teacher-student relationship to one of greater trust in which a student can thrive.

Art demonstrated this lesson in action to me, and without it, I don't know if I could have had a future in mathematics. This is something I will never forget, and I do my very best to pass it on to my students. It isn't always easy, especially being in academia, a locale where one is constantly under evaluation, and everyone faces individual, invisible struggles, but it is something that I always return to. Students should always feel they are more than the sum of their accomplishments, and no matter what, they are a valued part of my class. I always take time for my students, whatever the situation may be, and do my best to offer support, guidance, and advice, whenever appropriate.

2. TEACHING PHILOSOPHY AND COURSE DESIGN

I believe pedagogy, mentorship, and communication are some of the most direct ways for a mathematician to create an immediate and tangible positive impact in broader society. Because of this, I approach my

role as an educator and communicator of mathematics with pride, viewing it as one of the most significant components of my mathematical career.

I believe a mathematics education at any level should not only impart mathematical content, but also cultivate the following virtues and skills. First and foremost, mathematics presents a unique form of **abstract problem-solving** and **critical thinking**, skills which are applicable anywhere in one's life. Beyond developing these abilities, I want students to develop the **curiosity to explore problems** and **confidence to face them**. I aim to build **collaboration and community-building skills**: neither mathematics nor life are pursuits performed in isolation. I aim for students to build **patience** and **resilience** in the learning process. Students will often encounter concepts or questions they don't immediately get: I aim to teach acceptance of struggle, and patience to slowly gain understanding. Of course, this resilience complements many students' own resilience arising from their lived experiences. Finally, I hope to impart the lesson of **grace**, as mentioned earlier: students should always feel accepted and welcome, and know that their achievements do not define them.

I've been fortunate to have many opportunities to be an instructor of record for both upper- and lower-division mathematics courses. During my graduate studies at the University of California, Santa Cruz, I was a Graduate Teaching Fellow for seven quarters, which allowed me to be the instructor of record for many upper-division mathematics courses:

- Introduction to Partial Differential Equations;
- Introduction to Proofs and Problem Solving;
- Introduction to Number Theory (x2);
- Cryptography;
- Proofs and Problem Solving;
- Combinatorics.

On the other hand, as a Limited Term Assistant Professor (postdoc) at the University of Georgia, I have instructed Precalculus and Calculus I, adding breadth to my teaching experience. In the spring semester, I will instruct a graduate seminar on modular representation theory, which will present new experiences to further develop as an instructor. All this, along with experiencing fantastic pedagogy as a student, has provided opportunities to reflect and reshape my teaching practices, an ongoing process that has seen me shift towards a much more active teaching style.

I structure my lectures to be highly engaging and interactive, with numerous breaks in my lecturing to have students work out examples, or possibly even key results or theorems, themselves, in groups. I've found that this active-learning approach keeps students motivated and allows room for inquiry. Additionally, allocating time for small amounts of productive struggle in class can build confidence and understanding. It is important to stress the fact that mathematics is a collective, community-driven endeavor, and I reinforce that value by encouraging group work. Often, I find that by the end of a semester, my classes have become true learning communities, and many students maintain their bonds beyond the class. Finally, I understand that my students come from a diverse set of backgrounds and lived experiences, and increased interaction on all fronts helps bridge those gaps.

Moreover, I aim to share the **cultural** and **human aspects of mathematics** in my courses. Throughout, I will provide small extra credit opportunities by sharing readings about the history, philosophy, and modern viewpoints on mathematics, or given the wealth of mathematics communicators that have arisen in the past decade, I will ask students to watch videos on various extracurricular mathematical topics. In some instances, I have had students handwrite their reflections and questions, and submit them, and in others, I've had them meet with me and discuss the material in person. I've found students have been quite interested in these explorations (and not just because it's extra credit), with many of them telling me that they've never seen this side of mathematics, and its connection to our humanity.

For some courses I have taught, such as Math 101 - Proofs and Problem Solving at UC Santa Cruz, I've designed the course curriculum from the ground up. Math 101 served as an advanced problem solving course, teaching students to work through more difficult proof-based problems. Enrollment for the class was capped at 15 students, and students entered with a wide range of backgrounds and majors.

To achieve the goals I've set out for my students, a considerable amount of time was spent on having students work as teams to approach difficult problems requiring lateral thinking, a guided, productive struggle. I would also have students present proofs of problems on the board in front of their peers, clearly

explaining their proof and their problem-solving process. Assessment was a balanced mixture of individual and group work, allowing students to benefit from each other's experiences and strengths while still measuring individual progress.

The end result was a course where students achieved many of the goals I had set forth: they observably improved their problem solving skills, mathematical confidence, communication, and collaboration skills. Student feedback included:

- "Lectures were engaging, could tell Professor Sam loved what he was teaching."
- "I really liked how the class was structured so that homework was done in class, really minimized stress outside of class while simultaneously supporting learning and engagement. I also liked that the topics were taught in a way that someone like me without a huge math background could still solve a lot of the problems because we were taught several approaches and how to know when to use which approach by identifying patterns."
- "I just wanted to send a message saying thank you for such a great class. Math 101 has been one of my favorite classes, and you were a great teacher."

Since then, a number of those students have gone on to their graduate studies in mathematics or mathematics-adjacent fields. Some students asked me to write letters of recommendation for them as well, which I was happy to do.

3. MENTORSHIP AND OUTREACH

I've had wonderful mentorship during my mathematical career, and strive to serve that same role for the next generation of problem-solvers. I've participated numerous times in directed reading programs as both a graduate student and postdoc, helping undergraduates get a first taste of research math. These experiences allow students to take a more active role in their mathematical journey, where they develop independence and courage to venture into deeper mathematical waters. Topics I've guided students through have included elliptic curves and modular forms and the role of representation theory in particle physics. I approach the topics we learn with rigor while beginning to paint the bigger picture mathematically, ensuring the students have solid fundamentals while giving them the chance to explore and ask their own questions. In the future, I hope to not only run directed reading programs, but mentor undergraduate and graduate students through proper research.

One unusual instance of mentorship began when I gave a talk at the University of Washington in 2025. I received an email out of the blue from an undergraduate student interested in algebraic geometry, Precious Itsukor, visiting from Nigeria, asking if he could come see my talk. He came, and afterwards we chatted about a few topics. Afterwards, he'd occasionally email me asking me follow-up and career questions, as he aimed to go to graduate school in mathematics. For a year afterwards, I guided him through the process of getting into an REU and applying for graduate schools, though unfortunately, the changing political situation in the United States was a substantial difficulty.

Not only do I regularly give research talks, a necessary part of any research mathematician's career, I also find joy in giving expository or recreational talks to non-mathematical audiences. I take a special joy in imparting positive formative mathematical experiences, as some of my early formative mathematical experiences came from such talks, such as the DVD lecture from Arthur Benjamin, (the same Art mentioned previously) "The Joy of Mathematics."

I've given talks introducing undergraduates or early-career graduates to topics in representation theory and combinatorics. For instance, when I was invited in April 2024 to give a talk at the University of San Francisco mathematics colloquium, where I motivated the notion of a representation both abstractly and with its applications to other fields of science. I've additionally given talks on recreational mathematics and mathematical games to the general public and to audiences with scientific backgrounds outside mathematics, such as at a public lecture series in Santa Cruz. Moreover, I've volunteered at and attended numerous local math circles. These engagement opportunities are critical for increasing general mathematical participation, and I take great pride in being entertaining, informative, and interesting to a wide array of mathematical backgrounds.

APPENDIX A. ON GENERATIVE AI IN EDUCATION

There is of course an elephant in the room in the form of access to generative AI tools available to students. I am not professing to have a solution to this situation, but I will at least describe what I have done in the past year to adapt. The question which looms is: how can we, as professors, assess student comprehension while still leaving room for collaboration and exploration?

In the past, for upper- and lower-division courses, I gave non-examination-based assignments (homework and projects) significant weight on the final grade. In addition, I would assign open-ended projects to create a space for students to dive deeper into specific topics and stretch their creativity.

Unfortunately, I have temporarily moved away from this format. I, like many instructors, have observed a rise in homework grades and a drop in exam grades and student ability to navigate challenging problems. What this data suggests is that some students are outsourcing much of their work to generative AI. While I can sympathize with the students because the temptation is strong, this approach does not advance the goals I have set forth. Therefore, at least for now, I have set up an incentive structure which prioritizes problem-solving and critical thinking development, and discourages cognitive offloading, by switching, at least temporarily, to a more traditional, predominantly examination-based format. Though my courses now have examinations more often, I give lenient time limits, allowing for a reduced-stress environment.

I recognize this is a band-aid fix for a much larger issue, and supplement it with activities that stress the humanity of mathematics to my students. The practice I described earlier, sharing readings and videos on the history, philosophy, or modern practice of mathematics, started as a counterbalance to the heavily examination-based course format I've ran recently. The handwriting or in-person discussion I have found seems to encourage students to not just pass off the work to AI, and students have reacted positively.

Additionally, I discuss with students the evidence-based positives and negatives of generative AI usage. I believe students are fully aware of the downsides of using generative AI inappropriately and do care. Students have confided in me that they feel they must use generative AI in order to keep up with their peers, or lack the confidence to work without those tools. In situations like these, it is especially important to extend grace to these students. Often, they feel bad about their usage and know it is damaging to their education, but are trapped. Here, instructor support is pivotal to helping these students out of these holes and back on their feet. On the other hand, I do not tell students to abstain from AI usage in general, and instead, promote productive uses, such as generating additional problem sets with step-by-step solution guides. However, I also stress that AI interaction cannot replace human interaction, and that there are additional benefits from working through things together.

I find that AI tools can benefit course organization as well. For larger classes where individual feedback is more difficult to give, I think AI will eventually become quite helpful. I have participated in a pilot program from Edfinity featuring an AI grader for written homework assignments. At present I find the feedback is still subpar for the course experience I want to present, but I expect it to improve in time.

I cannot predict what the future of higher education will look like; we are in an era of rapid change, one which no one was prepared for. However, as things change, and we gain a better understanding of best teaching practices in the AI age, I will be happy to adapt and change my assessment style to best suit my students in alignment with the goals I have laid out.